SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that f be of bounded variation on $[a,b]$, if $x \in [a,b]$, let $V(x) = vf(a,x)$ and $pvtV(a) = 0$. Then every point of continuity of f is also a point of continuity of v. The converse is also true.

17. Prove that $f \in R(\alpha)$ on $[a,b]$ and assume that α has a continuous derivation α' on $[a,b]$. Then the Riemann integral $\int_a^b f(x)\alpha'(x)dx$ exists and we have

$$\int_a^b f(x)d\alpha(x) = \int_a^b f(x)\alpha'(x)dx.$$

18. Show that α is of bounded variation on $[a,b]$, ten $v(x)$ denote the total variation of α on $[a,x]$, if $a < x \le b$, and Let $v(a) = 0$. Let f be defined and bounded on $[a,b]$. If $f \in R(\alpha)$ on $[a,b]$, then $f \in R(V)$ on $[a,b]$.

19. State and prove cauchy condition for products.

20. State and prove Dirichlet's test for uniform convergence.

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APRIL/MAY 2023

GMA12 — REAL ANALYSIS – I

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define total variation.

2. Prove that f is monotonic on $[a,b]$, then f is bounded variation on $[a,b]$.

3. Define step function.

4. State Euler's summation formula.

5. State change of variable in Riemann integral.

6. State second mean-value theorem for Rimann-stieltjes integrals.

7. Define absolute and conditional convergence.

8. Define comparision test.

9. State Dirichlet's test for uniform convergence.

10. Define mean convergence.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Prove that f be of bounded variation on $[a,b]$, let v be defined on $[a,b]$ as follows $V(x) = V(a,x)$ if $a < x \leq b$, $V(a) = 0$. Then

(i) V is an increasing function $[a,b]$

(ii) $V - f$ is an increasing function $[a,b]$

Or

(b) Prove that f is of bounded variation on $[a,b]$, say $\Sigma / \Delta f_k$ for all partitions of $[a,b]$ then f is bounded on $[a,b]$. In fact $|f(x)| \leq |f(a)| + M$ for all x in $[a,b]$.

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12. (a) Prove that $f \in R(\alpha)$ and $g \in R(\alpha)$ on $[a,b]$ then $c_1 f + c_2 g \in R(\alpha)$ on $[a,b]$ (for any two constants c_1 and c_2) and we have $\int_a^b (c_1 f + c_2 g)\alpha\alpha = c_1 \int_a^b f d\alpha + c_2 \int_a^b g d\alpha$.

Or

(b) Prove that $f \in R(\alpha)$ on $[a,b]$ then $\alpha \in R(f)$ on $[a,b]$ and we have

$$\int_a^b f(x) d\alpha(x) + \int_a^b \alpha(x) df(x) = f(b)d(b) - f(a)\alpha(a)$$

13. (a) Prove that f is continuous on $[a,b]$ and if α is bounded variation on $[a,b]$ then $f \in R(\alpha)$ on $[a,b]$.

Or

(b) State and prove first mean value theorem for Riemann-stieltjes integral.

14. (a) Prove that Σ an be a conditionally convergent series with real valued terms. Let x and y be given members in the closed interval $[-\infty, +\infty]$, with $x \leq y$ there exists a rearrangement $\sum b_n$ of Σ an such that $\liminf\limits_{n\to\infty} t_n = x$ and $\limsup\limits_{n\to\infty} t_n = y_n$ where $t_n = b_1 + \cdots + b_n$.

Or

(b) Prove that $\lim p,q \to \infty f(p,q) = a$. For each fined p, assume that the limit $\lim p,q \to \infty (p,q)$ exists. Then the $\lim p \to q(\lim q \to \infty f(p,q))$. Then first is exists and has the value a.

15. (a) Prove that $f_n \to f$ uniformly on S. If each f_n is continuous at a point c of s, then the limit function f is also continuous at c.

Or

(b) Show that $\lim_{n\to\infty} f_n = f$ and $\lim_{n\to\infty} g_n = g$ on $[a,b]$ Define $h(x) = \int_a^x f(t) g(t) dt$, $h_n(x) = \int_a^x f_n(t) g_n(t) dt$, if $x \in [a,b]$, Then $h_n \to h$ uniformly on $[a,b]$.